Kuwait University
Faculty of Science
Department of Mathematics

Abstract Algebra II

0410-262
Second Exam

Monday, November 12, 2018
Fall 2018/19

Student Name											إسم الطـــالب
Student ID Number											الرقم الجامعي للطـــالب
											الرقم التسلسلي Serial Number

Section No. رقم الشعبة	Instructor Name أستاذ المقرر

Instructions to students | تعليمات للطالب

Time allowed: 1.25 hours. | وقت الإختبار: ساعة وربع.

This exam contains 4 questions. | يحتوي هذا الإختبار على 4 أسئلة.

ممنوع دخول الآلات الحاسبة أو أي وسيلة للإتصال داخل قاعة الإختبار.
Calculators and communication devices are not allowed in the examination room.

Question 1	
Question 2	
Question 3	
Question 4	
Total	

1. (2+1 pts.) Let $f(x) = 2 + 2x$ and $g(x) = 2 + 3x - x^2$ in $\mathbb{Z}_5[x]$.

(a) Compute: $f(x)\,g(x)$ in $\mathbb{Z}_5[x]$.

(b) Compute: $10\,(\,f^2(x) - g(x)\,)$ in $\mathbb{Z}_5[x]$.

2. (3+2+1 pts. each) In $\mathbb{Z}_5[x]$:

(a) Divide $f(x) = 2x^4 + x^2 - x + 1$ by $g(x) = 2x - 1$.

(b) Use the Remainder Theorem to determine the remainder when $h(x) = 2x^5 - 3x^3 + 2x + 1$ is divided by $x - 2$.

(c) Is -3 a root for $h(x)$ from part (b)? Explain.

3. (3+2+2 pts.)

(a) Determine whether $f(x) = x^3 + x + 1$ is irreducible over $\mathbb{Z}_3$. If $f(x)$ is reducible, then express it as a product of irreducible polynomials in $\mathbb{Z}_3[x]$.

(b) Use Eisenstein's Irreducibility Criterion to show that $f(x) = x^2 + 8x - 2$ is irreducible over $\mathbb{Q}$.

(c) Show that for a prime p, $f(x) = x^p + a \in \mathbb{Z}_p[x]$ is not irreducible for any $a \in \mathbb{Z}_p$.

4. (3 pts. each)

(a) Show that if F is a field, then F has no ideals other than (0) and F.

(b) If R is a commutative ring and $a \in R$, show that $I_a = \{\, x \in R :\, ax = 0 \,\}$ is an ideal of R.

(c) Let $\theta : R \to S$ be a ring homomorphism. Show that if $\ker\theta = \{\, 0_R \,\}$, then θ is one-to-one.