Kuwait University
Faculty of Science
Department of Mathematics

Abstract Algebra I

0410-261

Final (On-Line) Exam

Monday, September 28, 2020
Spring 2019/2020

Name										
ID Number										

Duration **2** hours [11:00 - 13:00] (This exam contains **5** questions).

Section No.	Instructor Name
1	**Dr. Abdullah Alazemi**

Give full reasons for your answer and State clearly any Theorem you use.
You are not allowed to open books/notes or any similar resources during the exam.

Question 1	/8
Question 2	/5
Question 3	/5
Question 4	/12
Question 5	/12
Total	/42

Good Luck

1. (**2+3+3 pts.**) Let G be a group of a finite order.

(a) Show that G has a unique identity.

(b) Show that if H and K are two subgroups of G, then $|\,H \cap K\,|$ divides $|\,G\,|$.

(c) Show that if G is a non-abelian group of order 10, then G hsa an element of order 5.

2. (**2+3 pts.**)

(a) List the isomorphism class representatives of abelian groups of order 100.

(b) Let G and H be two gourps. Show that if $\theta : G \to H$ is a group homomorphism with $\ker(\theta) = \{\, e_G \,\}$, then θ is one-to-one.

3. (**2+3 pts.**) Let H be a subgroup of a finite group G with $[G : H] = 2$

(a) Show that H is a normal subgroup of G.

(b) Compute the order of the quotient group G/H.

4. (**3 pts. each**) Let G and H be two isomorphic groups.

(a) Show that if $\theta : G \to H$ is an isomorphism then $\theta(G) \leq H$.

(b) Show that if G is abelian, then H is abelian as well.

(c) Show that if $f : G \to H$ is an isomorphism and B is a normal subgroup of H, then the subgroup $A = f^{-1}(B)$ of G is also normal in G.

(d) Show that if $|\,G\,| = p^2$ (p is prime number), then G has at least one subgroup of order p.

5. (**3 pts. each**) Let $GL_n(\mathbb{R}) = \{\,\text{all } n \times n \text{ nonsingular matrices with real entries}\,\}$ be a group with the operation of matrix multiplication.

(a) Show that $SL_n(\mathbb{R}) = \{\, A \in GL_n(\mathbb{R}) : |\,A\,| = 1 \,\}$ is a subgroup of $GL_n(\mathbb{R})$.

(b) Find the centralizer of $X = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ in $GL_2(\mathbb{R})$, denoted as $C\,(X)$.

(c) Show that $\theta : GL_n(\mathbb{R}) \to \mathbb{R}^*$ defined by $\theta(A) = |\,A\,|$ for each $A \in GL_n(\mathbb{R})$ is a homomorphism onto $\mathbb{R}^*$.

(d) Use the Fundamental Homomorphism Theorem to show that $GL_n(\mathbb{R})/SL_n(\mathbb{R})$ is isomorphic to $\mathbb{R}^*$.