1. (3+2 pts.)

(a) Let $\mathcal{A} = \{A_i : i \in \mathcal{I}\}$ be an indexed family of sets. Show that $\widetilde{\bigcup_{i \in \mathcal{I}} A_i} = \bigcap_{i \in \mathcal{I}} \widetilde{A_i}$.

(b) Let $\mathcal{U} = \mathbb{N}$ and $\mathcal{I} = \mathbb{N}$. Define $A_i = \mathbb{N} - \{1, 2, \cdots, i\}$ for all $i \in \mathcal{I}$. Find: $\bigcup_{i \in \mathcal{I}} \widetilde{A_i}$.

<u>Solution</u>:

2. (3+2 pts.)

(a) Show that for all $n \in \mathbb{N}$,

$$1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}.$$

(b) Express the terms of $(2x - 4yz^2)^5$ for $x, y, z \in \mathbb{R}$.

Solution:

3. **(3+3+2 pts.)**

(a) Let A, B and C be sets. Let $\mathcal{R} \subseteq A \times B$, $\mathcal{S} \subseteq B \times C$. Show that $(\mathcal{S} \circ \mathcal{R})^{-1} = \mathcal{R}^{-1} \circ \mathcal{S}^{-1}$.

(b) Let $m \neq 0$ be a fixed integer. Show that the relation $\equiv_m$ is an equivalence relation on $\mathbb{Z}$.

(c) Let $\mathcal{R}$ be a relation on a nonempty set A. Prove that $\mathcal{R} \cup \mathcal{R}^{-1}$ is symmetric.

Solution:

4. (**3+2+2 pts.**) Let A and B be two nonempty sets and $f : A \to B$ is a bijection mapping.

(a) Show that f^{-1} is a function from B to A.

(b) Show that f^{-1} is one-to-one.

(c) Show that f^{-1} is a bijection from B to A.

Solution: