Kuwait University

Faculty of Science

Department of Mathematics

Math 250
Foundations of Mathematics
Summer 2022/2023

Final Exam
July 22, 2023

Name											Serial Number
ID Number											

Duration **2** hours (This exam contains **5** questions).

Section No.	Instructor Name
1	**Dr. Abdullah Alazemi**

Give full reasons for your answer and State clearly any Theorem you use.

Question 1	
Question 2	
Question 3	
Question 4	
Question 5	
Total	40

1. (**6 pts.**)

(a) Prove or disprove: For any two sets A and B, if $A \times B = \phi$, then A or B is the emptyset.

(b) Let $\mathcal{R}$ be a relation on $\mathbb{N}$ so that $a\mathcal{R}b \Leftrightarrow a \mid b$ for all $a, b \in \mathbb{N}$. Determine if $\mathcal{R}$ is an antisymmetric relation on $\mathbb{N}$.

2. (**7 pts.**)

(a) Let $\mathcal{R}$ be some relation on a nonempty set A, and let $\mathcal{S}$ be a transitive relation containing $\mathcal{R}$. Show that $\mathcal{R} \circ \mathcal{R} \subseteq \mathcal{S}$.

(b) Let $f : A \to B$ and $g : B \to C$ be two functions for some nonempty sets A, B, and C. Without assuming that $g \circ f$ is a function, show that if $(x, z_1), (x, z_2) \in g \circ f$, then $z_1 = z_2$, where $x \in A$ and $z_1, z_2 \in C$.

3. (**8 pts.**) Let $\theta : \mathbb{N} \times \mathbb{N} \to \mathbb{N}$ be a bijection defined by $\theta((m, n)) = 2^{m-1}(2n-1)$. You may use the fact: $A \approx C$ and $B \approx D$ implies $A \times B \approx C \times D$.

(a) Show that if A and B are two denumerable sets, then $A \times B$ is a countable set.

(b) What is the cardinality of $A \times B$?

(c) Find the inverse image $\theta^{-1}(Y)$, where $Y = \{3, 12\}$.

4. (**9 pts.**)

(a) Let $f : \mathbb{N} \times \mathbb{N} \to \mathbb{N}$ be a function given by $f(a, b) = a + b$ for all $a, b \in \mathbb{N}$. Decide whether f is one-to-one and onto $\mathbb{N}$.

(b) Let $f : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $f(a, b) = (a + b, a - b)$. Is f a bijection? Explain.

5. (**10 pts.**)

(a) Let $a, b, c, d \in \mathbb{R}$ so that $a < b$ and $c < d$. Let f denote the relation from (a, b) to (c, d) defined for any $x \in (a, b)$ by

$$f(x) = \frac{d-c}{b-a}(x-a) + c.$$

Determine if f is onto function.

(b) Let $A = (1, 2) \cup [5, 7)$. Provide a function g that can be used to show that $A \approx (0, 1)$. **Do not prove that g is a bijection**.