Kuwait University
Faculty of Science
Department of Mathematics

# Abstract Algebra II

# 0410-262
# Second Exam

**Monday, November 12, 2018**
**Fall 2018/19**

| Student Name | | | | | | | | | | | إسم الطـــالب |
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| Student ID Number | | | | | | | | | | | الرقم الجامعي للطـــالب |
| | | | | | | | | | | | الرقم التسلسلي<br>Serial Number |

| Section No. رقم الشعبة | Instructor Name أستاذ المقرر |
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Instructions to students — تعليمات للطالب

Time allowed: 1.25 hours. — وقت الإختبار: ساعة وربع.

This exam contains 4 questions. — يحتوي هذا الإختبار على 4 أسئلة.

ممنوع دخول الآلات الحاسبة أو أي وسيلة للإتصال داخل قاعة الإختبار.

Calculators and communication devices are not allowed in the examination room.

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| Question 1 | |
| Question 2 | |
| Question 3 | |
| Question 4 | |
| Total | |

**1.** (**2+1 pts.**) Let $f(x) = 2 + 2x$ and $g(x) = 2 + 3x - x^2$ in $\mathbb{Z}_5[x]$.

(a) Compute: $f(x)\,g(x)$ in $\mathbb{Z}_5[x]$.

(b) Compute: $10\,(\,f^2(x) - g(x)\,)$ in $\mathbb{Z}_5[x]$.

**2.** (**3+2+1 pts. each**) In $\mathbb{Z}_5[x]$:

(a) Divide $f(x) = 2x^4 + x^2 - x + 1$ by $g(x) = 2x - 1$.

(b) Use the Remainder Theorem to determine the remainder when $h(x) = 2x^5 - 3x^3 + 2x + 1$ is divided by $x - 2$.

(c) Is $-3$ a root for $h(x)$ from part (b)? Explain.

**3. (3+2+2 pts.)**

(a) Determine whether $f(x) = x^3 + x + 1$ is irreducible over $\mathbb{Z}_3$. If $f(x)$ is reducible, then express it as a product of irreducible polynomials in $\mathbb{Z}_3[x]$.

(b) Use Eisenstein's Irreducibility Criterion to show that $f(x) = x^2 + 8x - 2$ is irreducible over $\mathbb{Q}$.

(c) Show that for a prime $p$, $f(x) = x^p + a \in \mathbb{Z}_p[x]$ is not irreducible for any $a \in \mathbb{Z}_p$.

**4.** (**3 pts. each**)

(a) Show that if $F$ is a field, then $F$ has no ideals other than $(0)$ and $F$.

(b) If $R$ is a commutative ring and $a \in R$, show that $I_a = \{ x \in R : ax = 0 \}$ is an ideal of $R$.

(c) Let $\theta : R \to S$ be a ring homomorphism. Show that if $\ker \theta = \{ 0_R \}$, then $\theta$ is one-to-one.