
Give full reasons for your answer. State clearly any Theorem you use.

1. (4+4 pt) Choose only two problems from the following:

- a. (4pt) Use a truth table to show that " $(P \Rightarrow Q) \iff (\sim Q \Rightarrow \sim P)$ " is a *tautology*.
- b. (4pt) Find a *denial* for " $(\forall y)(\exists x)[(x > y + 1) \vee [(x = y + 1) \wedge (xy > 0)]]$ ".
- c. (4pt) Let $x, y \in \mathbb{R}$ such that $x < 2y$. Show that if $7xy \leq 3x^2 + 2y^2$, then $3x \leq y$.
"You may use a proof by contrapositive".
- d. (4pt) Let $\mathcal{U} = \mathbb{N}$. Define $A_i = \mathbb{N} - \{1, 2, \dots, i\}$ for all $i \in \mathbb{N}$. Find $\bigcap_{i \in \mathbb{N}} \tilde{A}_i$.

2. (4pt) Show that $(\forall n \in \mathbb{N}) [2 \mid (9^n - 5^n)]$.

3. (5pt) If any, find all integer solutions to the equation $3n - 5m = 2$.

4. (4pt) Let A and B be two sets. Show that $\widetilde{A \cup B} = \tilde{A} \cap \tilde{B}$. "Do not use Venn diagram".

5. (4pt) Let A and B be two sets. Show that $\mathcal{P}(A \cap B) = \mathcal{P}(A) \cap \mathcal{P}(B)$.

Bonus Question (1pt):

- Let A and B be two sets. Show that $A - B = A \cap \tilde{B}$.