
Give full reasons for your answer. State clearly any Theorem you use.

1. (3pt) Use a truth table to show that " $(P \Rightarrow Q) \iff (\sim Q \Rightarrow \sim P)$ " is a *tautology*.

2. (3pt) Find a *denial* for

$$(\forall z)(\exists x)(\exists y)[((x > z) \wedge (y > z)) \wedge \sim (\exists w)(x + y < w < xz)].$$

3. (3pt) Let $a, b, c \in \mathbb{Z}$ with $c = am + bn$ for some integers m and n . If $d = \text{GCD}(a, b)$, then $d \mid c$.

4. (4pt) Find all, if any, integer solutions to the equation $3m - 7n = 5$.

5. (4pt) Let A and B be two subsets of a given universe \mathcal{U} . Show that $A \subseteq B$ if and only if $\tilde{B} \subseteq \tilde{A}$.

6. (4pt) Let $\mathcal{U} = \mathbb{N}$. Define $A_i = \mathbb{N} - \{1, 2, 3, \dots, i\}$ for all $i \in \mathbb{N}$. Find $\bigcap_{i \in \mathbb{N}} \tilde{A}_i$.

7. (4pt) Show that for all $n \in \mathbb{N}$, $5 \mid (n^5 + 4n)$.

Bonus Question (1pt):

- Let $a, b, c \in \mathbb{Z}$. Show that if $a \mid b$ and $a \mid c$, then $a \mid (b - c)$.