
Give full reasons for your answer. State clearly any Theorem you use.

1. (3pt) Let a, b, d_1 , and d_2 be positive integers.
 - (a) State the definition of the least common multiple of a and b , denoted by $\text{LCM}(a, b)$.
 - (b) Show that if d_1 and d_2 are two least common multiples of a and b , then $d_1 = d_2$.
2. (3pt) Use a truth table to show that " $(P \Rightarrow Q) \iff (\sim Q \Rightarrow \sim P)$ " is a tautology.
3. (3pt) Show that $\sqrt{2}$ is an irrational number.
4. (3pt) Show that for all $n \in \mathbb{N}$, $5 \mid (9^n - 4^n)$.
5. (4pt) Find a denial for

$$(\forall z)(\exists y)(\exists x) [((y > z) \wedge (x > z)) \vee \sim (\exists w)[x + y < w < xz]].$$

6. (4pt) Find all integer solutions to the equation $3m - 7n = 5$.

Bonus Question (1pt):

- Let $a, b, c \in \mathbb{Z}$. Show that if $a \mid b$ and $a \mid c$, then $a \mid (b - c)$.