

Give full reasons for your answer. State clearly any Theorem you use.

1. (3pt) Let  $a, b, d_1$ , and  $d_2$  be positive integers.
  - (a) State the definition of the greatest common divisor of  $a$  and  $b$ , denoted by  $\text{GCD}(a, b)$ .
  - (b) Show that if  $d_1$  and  $d_2$  are two greatest common divisors of  $a$  and  $b$ , then  $d_1 = d_2$ .
2. (3pt) Use a truth table to show that " $\sim (P \Rightarrow Q)$ " is equivalent to " $P \wedge (\sim Q)$ ".
3. (3pt) Let  $a$  be a prime number, and let  $b$  and  $c$  be positive integers. Show that if  $a$  divides  $bc$ , then  $a$  divides  $b$  or  $a$  divides  $c$ .
4. (3pt) Show that for all  $n \in \mathbb{N}$ ,  $3 \mid (7^n - 4^n)$ .
5. (4pt) Let  $P(x)$  be an open sentence for a given variable  $x \in \mathcal{U}$ , for some  $\mathcal{U}$ . Find a denial for " $(\exists!x)P(x)$ ".
6. (4pt) Find all integer solutions to the equation  $3m - 7n = 5$ .

**Bonus Question (1pt):**

- Let  $a, b, c \in \mathbb{Z}$ . Show that if  $a \mid b$  and  $a \mid c$ , then  $a \mid (b - c)$ .