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Give full reasons for your answer. State clearly any Theorem you use.

1. (3pt) Use a truth table to show that " $\sim (P \Rightarrow Q) \iff (P \wedge \sim Q)$ " is a *tautology*.
2. (3pt) Find a *denial* for  $(\exists!x)P(x)$ .

Hint: use  $(\exists!x)P(x) \equiv (\exists x)[P(x) \wedge (\forall y)[P(y) \Rightarrow x = y]]$ .

3. (3pt) Let  $x, y \in \mathbb{R}$  such that  $x < 2y$ . Show that if  $7xy \leq 3x^2 + 2y^2$ , then  $3x \leq y$ .  
[You may use a proof by contradiction].
4. (4pt) Find all, if any, integer solutions to the equation  $2m + 3n = 7$ .
5. (4pt) Show that  $\widetilde{A \cap B} = \widetilde{A} \cup \widetilde{B}$ , for any two sets  $A$  and  $B$  of a given universe  $\mathcal{U}$ .
6. (4pt) Let  $\mathcal{U} = \mathbb{N}$ . Define  $A_i = \{2i + 1\}$  for all  $i \in \mathbb{N}$ . Find  $\bigcap_{i \in \mathbb{N}} \widetilde{A}_i$ .
7. (4pt) Show that for all  $n \in \mathbb{N}$ ,  $1 + 3 + 5 + \cdots + (2n - 1) = n^2$ .

**Bonus Question (1pt):**

- Express the terms of  $(1 - 2x)^4$  for  $x \in \mathbb{R}$ .