

Give full reasons for your answer. State clearly any Theorem you use.

1. (1+2pt) Let  $A = \{1, 2, \{3, 4\}\}$ .

- (a) State the definition of the power set of a given set.
- (b) Find the power set of  $A$ .

2. (3pt) Let  $A$  and  $B$  be two sets. Show that

$$\mathcal{P}(A \cap B) = \mathcal{P}(A) \cap \mathcal{P}(B).$$

3. (3pt) Let  $A, B$ , and  $C$  be three sets. Show that

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C).$$

4. (3pt) Show that for all natural numbers  $n$ ,

$$\sum_{i=1}^n i = \frac{n(n+1)}{2}.$$

5. (4pt) Show that for all natural numbers  $n$ , 3 divides  $n^3 + 5n + 6$ .

6. (4pt) Let  $\mathcal{A}$  be a family of sets. Show that

$$\widetilde{\bigcup_{A \in \mathcal{A}} A} = \bigcap_{A \in \mathcal{A}} \tilde{A}.$$

**Bonus Question (1pt):**

- For each  $i \in \mathbb{N}$ , let  $A_i = \{j \in \mathbb{Z} : -i < j < i\}$ . Find  $\bigcup_{i \in \mathbb{N}} \tilde{A}_i$ .