Kuwait University
First Midterm
Math 261 - Abstract Algebra

Mathematics Department
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Give full reasons for your answer. State clearly any Theorem you use.

1. (3 pts.) Let $*$ be defined on $\mathbb{Z}$ by $m * n = 2^{mn}$ for all $m, n \in \mathbb{Z}$. Is $*$ a binary operation on $\mathbb{Z}$? Is it associative? Is it commutative? Explain you answers.

2. (3 pts.) Let $f : A \to B$ and $g : B \to C$ be two onto mappings. Show that $g \circ f : A \to C$ is also onto mapping.

3. (4 pts.) Let $G = S_8$.

(a) Compute $(1\ \ 3\ \ 7)(2\ \ 4\ \ 5\ \ 6)^{-1}(1\ \ 8\ \ 7)^{-1}(2\ \ 4\ \ 5\ \ 6)$ in G.

(b) Decide whether $\alpha = (1\ \ 5\ \ 8)(2\ \ 4)(3\ \ 6\ \ 7)$ is even or odd in G.

4. (3+2 pts.) Let G be a permutation group on a nonempty set S and $T \subseteq S$.

(a) Show that G_T, the element-wise stabilizer, is a subgroup of G.

(b) Assume that $S = \{1, 2, 3, 4\}$, $G = Sym(S) = S_4$ and $T = \{3\}$. Compute $G_{(T)}$, the set-wise stabilizer.

5. (2+3 pts.) Let G be abelian group with operation $*$ and let a be an element of G.

(a) Find $C(a)$, the centralizer of a in G.

(b) Let $GL_2(\mathbb{R})$ be the group of all 2×2 nonsingular matrices with real entries under matrix multiplication. Compute $C\left(\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}\right)$.

Bonus Question (1pt):

- Write $(3\ \ 4\ \ 5)\ (1\ \ 2\ \ 5\ \ 4)\ (1\ \ 3\ \ 5)$ as a single cycle or a product of disjoint cycles.