Kuwait University
First Midterm
Math 250 - Foundations of Math

Mathematics Department
March 9, 2015
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Give full reasons for your answer. State clearly any Theorem you use. Do not use Venn diagrams to prove.

1. (9pt) **Choose only <u>three</u> problems from a, b, c, and d**:

   a. (3pt) Use a truth table to show that "$\big[P \wedge (P \Rightarrow Q)\big] \Rightarrow Q$" is a *tautology.*

   b. (3pt) Find a *denial* for "$(\exists! x)\big[P(x)\big]$" for some open sentence $P(x)$ with x is in some universe.

   c. (3pt) Let $x, y \in \mathbb{Z}$. If x and y are both odd, then xy is odd.

   d. (3pt) Let $\mathcal{U} = \mathbb{N}$. Define $A_i = \mathbb{N} - \{1, 2, \cdots, i\}$ for all $i \in \mathbb{N}$. $\bigcap_{i \in \mathbb{N}} \widetilde{A}_i$.

2. (3pt) Let $A, B,$ and C be three sets. Show that $(A \cup C) - B = (A - B) \cup (C - B)$.

3. (4pt) Let A and B be two nonempty sets. Show that if $A \subseteq B$, then $\widetilde{B} \subseteq \widetilde{A}$.

4. (4pt) Let $\mathcal{M}$ be a family of sets. Show that $\widetilde{\bigcup_{A \in \mathcal{M}} A} = \bigcap_{A \in \mathcal{M}} \widetilde{A}$.

5. (5pt) If any, find all integer solutions to the equation $3m - 4n = 2$.

Bonus Question (1pt):

- Let $A = \{1\}$ and $B = \{1, 2\}$. Is $\mathcal{P}(A) \cup \mathcal{P}(B) = \mathcal{P}(A \cup B)$. Explain.