

Give full reasons for your answer. State clearly any Theorem you use.

1. (3pt) Use a truth table to show that " $\sim (P \Rightarrow Q) \iff (P \wedge \sim Q)$ " is a *tautology*.
2. (3pt) Find a *denial* for $(\exists!x)P(x)$.
3. (3pt) Let $a, b, c \in \mathbb{Z}$. Show that if a divides b and a divides c , then a divides $b - c$.
4. (4pt) Find all, if any, integer solutions to the equation $2m + 3n = 7$.
5. (4pt) Show that $\widetilde{A \cap B} = \widetilde{A} \cup \widetilde{B}$, for any two sets A and B of a given universe \mathcal{U} .
6. (4pt) Let $\mathcal{U} = \mathbb{N}$. Define $A_i = \{2i + 1\}$ for all $i \in \mathbb{N}$. Find $\bigcap_{i \in \mathbb{N}} \widetilde{A}_i$.
7. (4pt) Show that for all $n \in \mathbb{N}$, $\sum_{i=1}^n i = \frac{n(n+1)}{2}$.

Bonus Question (1pt):

- Express the terms of $(1 - 2x)^4$ for $x \in \mathbb{R}$.