Kuwait University
Faculty of Science
Department of Mathematics

Abstract Algebra II

0410-262
First Exam

Monday, October 8, 2018
Fall 2018/2019

Student Name											إسم الطـــالب
Student ID Number											الرقم الجامعي للطـــالب
											الرقم التسلسلي Serial Number

Section No. رقم الشعبة	Instructor Name أستاذ المقرر
1	**Dr. Abdullah Alazemi**

Instructions to students — تعليمات للطالب

Time allowed: 1.25 hours. — وقت الإختبار: ساعة وربع.

This exam contains 4 questions. — يحتوي هذا الإختبار على 4 أسئلة.

ممنوع دخول الآلات الحاسبة أو أي وسيلة للإتصال داخل قاعة الإختبار.

Calculators and communication devices are not allowed in the examination room.

Give full reasons for your answer. State clearly any Theorem you use.

Question 1	
Question 2	
Question 3	
Question 4	
Total	

1. (2+2 pts.) Let R be a ring with the zero 0.

(a) Show that for all $a \in R$, we have $0a = 0$.

(b) Show that $a^2 - b^2 = (a + b)(a - b)$ for all $a, b \in R$ if and only if R is commutative.

Solution:

2. (1+2+3 pts.)

(a) Find all zero divisors in $\mathbb{Z}_6$.

(b) Solve the equation $x^2 - 4 = 0$ in $\mathbb{Z}_5$.

(c) Let p be a prime. Show that in the ring $\mathbb{Z}_p$, we have $(a+b)^p = a^p + b^p$ for all $a, b \in \mathbb{Z}_p$.

Solution:

3. (3+4 pts.) Let R be a ring.

(a) The center of R is defined by $Z(R) = \{x \in R : ax = xa \text{ for all } a \in R\}$. Show that $Z(R)$ is a subring of R.

(b) Prove that if C denotes any collection of subrings R, then the intersection of all of the rings in C is also a subring of R. What about the union of subrings of R?

Solution:

4. (2+2+4 pts.)

(a) Show that a division ring contains exactly two idempotent elements.

(b) Let $R = \left\{ a + b\sqrt{2} : a, b \in \mathbb{Q} \right\}$ and $S = \left\{ a + b\sqrt{3} : a, b \in \mathbb{Q} \right\}$ be two subfields of $\mathbb{R}$. Verify that $\theta : R \to S$ defined by $\theta\left(a + b\sqrt{2} \right) = a + b\sqrt{3}$ is not a ring homomorphism.

(c) Suppose that R and S are two isomorphic rings and that R is an integral domain. Show that S is integral domain by showing that S is commutative ring with no zero divisors and with a unity.

Solution: