Kuwait University | **Mathematics Department**
Final Exam Revision | **December 15, 2013**
Math 250 - Foundations of Math | **Instructor: A. AlAzemi**

Give full reasons for your answer. State clearly any Theorem you use.

1. For any propositions P and Q, show that "$P \Rightarrow Q$" is equivalent to "$(\sim Q) \Rightarrow (\sim P)$".
2. Use a contrapositive proof to show that if x and y are both odd integers, then xy is odd.
3. Let $P(x)$ be an open sentence with a variable x in some universe $\mathcal{U}$. Show that
$$\sim (\forall x)[P(x)] \text{ is equivalent to } (\exists x)[\sim P(x)].$$
4. Find a denial for $(\exists! x)P(x)$, for any given proposition $P(x)$.
5. Show that there is no odd integer that can be expressed in the form $4m - 1$ and in the form $4n + 1$ for integers m and n.
6. Show that $[\ 5 \,\big|\, (9^n - 4^n)\ ]$ for any $n \in \mathbb{N}$.
7. If any, find all integers m and n so that $21n - 5m = 7$.
8. Let $A = \big\{1, \{2\}, \{3, 4\}\big\}$. Find $\mathcal{P}(A)$.
9. Let A and B be two non-empty sets in some universe $\mathcal{U}$.
   (a) Show that $A - B = A \cap \widetilde{B}$.
   (b) Show that $\widetilde{A \cup B} = \widetilde{A} \cap \widetilde{B}$.
10. Show that $\mathcal{P}(A \cap B) = \mathcal{P}(A) \cap \mathcal{P}(B)$ for any sets A and B.
11. For each $i \in \mathbb{N}$, let $A_i = \{j \in \mathbb{N} : j < i\}$. Find $\bigcup_{i \in \mathbb{N}} A_i$ and $\bigcap_{i \in \mathbb{N}} A_i$.
12. Let $x \in \mathbb{R}$ with $x \geq -1$. Use a proof by induction to show that $(1 + x)^n \geq 1 + nx$ for all $n \in \mathbb{N}$.
13. Let $\mathcal{R}$ be the relation on $\mathbb{Z}$ given by $x\mathcal{R}y \iff x - y$ is even. Show that $\mathcal{R}$ is an equivalence relation on $\mathbb{Z}$.
14. Let $m \neq 0$ be a fixed integer. Show that the relation "$\equiv_m$" is an equivalence relation on $\mathbb{Z}$.
15. Let $\mathcal{R}$ be a relation on $\mathbb{N}$ so that $a\mathcal{R}b \iff 3 \,\big|\, a + 2b$. Show that $\mathcal{R}$ is an equivalence relation on $\mathbb{N}$.
16. Let $\mathcal{R}$ be a relation on $\mathbb{N}$ so that $a\mathcal{R}b \iff a \,\big|\, b$ for all $a, b \in \mathbb{N}$. Show that $\mathbb{N}$ is a poset with respect to $\mathcal{R}$.
17. Let $f : \mathbb{N} \times \mathbb{N} \to \mathbb{N}$ be a function defined by $f\big((m, n)\big) = 2^{m-1}(2n - 1)$. Show that f is a bijection.
18. Let A, B, and C be three sets. Let $f : A \to B$, $g : B \to C$, and $g \circ f : A \to C$ be functions.
    (a) Show that if f and g are both surjections, then $g \circ f$ is also surjection.
    (b) Show that if $g \circ f$ is surjection, then g is also surjection.

(c) Show that if f and g are both injections, then $g \circ f$ is also injection.

(d) Show that if $g \circ f$ is injection, then f is also injection.

19. Let $f : A \to B$ be a function and let $\{X_i : i \in \mathcal{I}\} \subseteq A$ for some indexing set $\mathcal{I}$. Show that $f\left(\bigcup_{i \in \mathcal{I}} X_i\right) = \bigcup_{i \in \mathcal{I}} f(X_i)$.

20. Let $f : \mathbb{N} \times \mathbb{N} \to \mathbb{N}$ be a function defined by $f\big((m, n)\big) = 2^{m-1}(2n - 1)$. Find $f^{-1}\big(\{12, 16\}\big)$.

21. Let "$\approx$" be the equivalent relation on the class of all sets. Show that the relation "$\approx$" is an equivalence relation on the class of all sets.

22. Let $A \approx C$ and $B \approx D$ for some sets A, B, C, and D. Show that $A \times B \approx C \times D$.

23. Let $A = \left\{\frac{1}{2n+1} : n \in \mathbb{N}\right\}$.

    (a) State the definition of a countable set.

    (b) Show that A is countable.

24. Show that if A and B are two denumerable sets, then $A \times B$ is also denumerable.

25. Let $A = (3, 4) \cup [6, 7) \subseteq \mathbb{R}$.

    (a) Show that $A \approx (0, 1)$ without using the Horizontal line test.

    (b) What is the cardinal number of A.