Kuwait University | **Mathematics Department**
Second Midterm (Version B) | **December 8, 2014**
Math 250 - Foundations of Math | **Instructor: A. AlAzemi**

Give full reasons for your answer. State clearly any Theorem you use.

1. (3pt) Show that if $A \times B = \phi$, then $A = \phi$ or $B = \phi$.

2. (4pt) Let A, B, and C be three sets. Let $\mathcal{R} \subseteq A \times B$ and $\mathcal{S} \subseteq B \times C$ be two relations. Show that $(\mathcal{S} \circ \mathcal{R})^{-1} = \mathcal{R}^{-1} \circ \mathcal{S}^{-1}$.

3. (4pt) Let $\mathcal{R}$ be an equivalence relation on $\mathbb{Z}$. Show that for all $x, y \in \mathbb{Z}$, $x \,\mathcal{R}\, y$ if and only if $x/\mathcal{R} = y/\mathcal{R}$.

4. (4pt) Let $\mathcal{R}$ be a relation on $\mathbb{N}$ so that $a\mathcal{R}b$ if and only if $a + b$ is even and $a \leq b$ for all $a, b \in \mathbb{N}$. Show that $\mathbb{N}$ is a poset with respect to $\mathcal{R}$.

5. (5pt) Let $f_1 = 1$, $f_2 = 1$, and $f_{n+2} = f_{n+1} + f_n$ for all $n \in \mathbb{N}$. Show that f_{3n} is an even number for all natural number n.

6. (5pt) Let $\mathcal{R}$ be a relation on $\mathbb{N}$ so that $a\mathcal{R}b$ if and only if $3 \,\big|\, a + 2b$. Show that $\mathcal{R}$ is an equivalence relation on $\mathbb{N}$.

Bonus Question (1pt):

- Define a partial order $\mathcal{R}$ on $\mathbb{N} \times \mathbb{N}$ by $(a, b)\mathcal{R}(c, d)$ if and only if $\big[a < c \text{ or } (a = c \text{ and } b \leq d)\big]$. Is $(1, 3)\mathcal{R}(1, 2)$? Explain.