Kuwait University
Second Midterm (Version A)
Math 250 - Foundations of Math

Mathematics Department
December 3, 2014
Instructor: A. AlAzemi

Give full reasons for your answer. State clearly any Theorem you use.

1. (3pt) Let $\mathcal{R}$ be the relation on $\mathbb{Q}$ given by $\{(x, y) \in \mathbb{Q} \times \mathbb{Q} : x - y \in \mathbb{Z}\}$. Find the equivalence class of $\dfrac{1}{3}$, denoted by $\overline{\frac{1}{3}}$.

2. (4pt) Let A, B, and C be three sets. Show that $A \times (B \cap C) = (A \times B) \cap (A \times C)$.

3. (4pt) Let A, B, and C be three sets. Let $\mathcal{R} \subseteq A \times B$ and $\mathcal{S} \subseteq B \times C$ be two relations. Show that $(\mathcal{S} \circ \mathcal{R})^{-1} = \mathcal{R}^{-1} \circ \mathcal{S}^{-1}$.

4. (4pt) Let $\mathcal{R}$ be a relation on $\mathbb{N}$ so that $a\mathcal{R}b$ if and only if $a \big| b$ for all $a, b \in \mathbb{N}$. Show that $\mathcal{R}$ is a partial order relation on $\mathbb{N}$.

5. (5pt) Let $f_1 = 1$, $f_2 = 1$, and $f_{n+2} = f_{n+1} + f_n$ for all $n \in \mathbb{N}$. Show that f_{3n} is an even number for all natural number n.

6. (5pt) Let $\mathcal{R}$ be a relation on $\mathbb{N} \times \mathbb{N}$ so that $(a, b)\mathcal{R}(c, d)$ if and only if $ad = bc$. Show that $\mathcal{R}$ is an equivalence relation on $\mathbb{N} \times \mathbb{N}$.

Bonus Question (1pt):

- Define a partial order $\mathcal{R}$ on $\mathbb{N} \times \mathbb{N}$ by $(a, b)\mathcal{R}(c, d)$ if and only if $\big[a < c \text{ or } (a = c \text{ and } b \leq d)\big]$. Is $(1, 3)\mathcal{R}(1, 2)$? Explain.