
Give full reasons for your answer. State clearly any Theorem you use.

1. (3pt) Let $f : A \to B$ and $g : B \to C$ be two functions. Show that if $(a, c_1), (a, c_2) \in g \circ f$, then $c_1 = c_2$.

2. (3pt) Let $f : A \to B$ be a function. Show that if f^{-1} is a function, then f^{-1} is one-to-one.

3. (2pt for each part) Answer True or False: Give reasons for your answer.

(a) If $A \times B = \phi$, then A or B is empty.

(b) If $\mathcal{R}$ is the relation on $\mathbb{Z}$ given by $a\mathcal{R}b$ if and only if a divides b, then $\mathcal{R}$ is antisymmetric.

4. (4pt) Let "$\approx$" be the equivalent relation on the class of all sets. Show that the relation "$\approx$" is an equivalence relation on the class of all sets.

5. (4pt) Show that if A and B are two non-empty finite sets, then $A \cup B$ is finite.

6. (4pt) Let $A = \left\{ \dfrac{1}{2k+3} : k \in \mathbb{N} \right\}$. Show that A is countable.

7. (5pt) Let $f_1 = 1$, $f_2 = 1$, and $f_{n+2} = f_{n+1} + f_n$ for all $n \in \mathbb{N}$. Show that f_{3n+1} is an odd number for all natural number n.

8. (5pt) Show that $A = (1, 2) \cup [4, 5) \subseteq \mathbb{R}$ is uncountable.

9. (4+3+1 pt) Let $f : \mathbb{N} \times \mathbb{N} \to \mathbb{N}$ be a function defined by $f\big((m, n)\big) = 2^{m-1}(2n - 1)$.

(a) Show that f is a bijection.

(b) Find $f^{-1}(\{11, 32\})$.

(c) Show that $\mathbb{N} \times \mathbb{N}$ is a denumerable set.