Kuwait University **Mathematics Department**
Second Midterm **December 4, 2013**
Math 250 - Foundations of Math **Instructor: A. AlAzemi**

Give full reasons for your answer. State clearly any Theorem you use.

1. (3pt) Let $\mathcal{R}$ be a relation on a set A. Show that $\mathcal{R} \cup \mathcal{R}^{-1}$ is a symmetric relation.

2. (5pt) Let A, B, and C be sets. Let $\mathcal{R} \subseteq A \times B$ and $\mathcal{S} \subseteq B \times C$ be two relations. Show that $(\mathcal{S} \circ \mathcal{R})^{-1} = \mathcal{R}^{-1} \circ \mathcal{S}^{-1}$.

3. (5pt) Let $\mathcal{R}$ be a relation on $\mathbb{N}$ so that $a\mathcal{R}b$ if and only if $3 \,\big|\, x + 2y$. Show that $\mathcal{R}$ is an equivalence relation on $\mathbb{N}$.

4. (2+4pt) Let $f : A \to B$ and $g : B \to C$ be two functions.

(a) State the definition of a function.

(b) Show that if $(a, c_1), (a, c_2) \in g \circ f$, then $c_1 = c_2$.

5. (2+4pt) Let $f : A \to B$ be a function. Then,

(a) State the definition of a one-to-one function.

(b) Show that if f^{-1} is a function, then f^{-1} is one-to-one.

Bonus Question (1pt):

- Define a partial order $\mathcal{R}$ on $\mathbb{N} \times \mathbb{N}$ by $(a, b)\mathcal{R}(c, d)$ if and only if $\big[a < c \text{ or } (a = c \text{ and } b \leq d)\big]$. Is $(1, 3)\mathcal{R}(1, 2)$? Explain.