Kuwait University | **Mathematics Department**
Final Exam | **December 28, 2013**
Math 250 - Foundations of Math | **Instructor: A. AlAzemi**

Give full reasons for your answer. State clearly any Theorem you use.

1. (3pt) Let A and B be sets. Show that $A \not\subseteq B \iff A \cap \widetilde{B} \neq \phi$.

2. (3pt) Prove that if A and B are sets, then $\mathcal{P}(A \cap B) = \mathcal{P}(A) \cap \mathcal{P}(B)$.

3. (3pt) Show that there is no odd integer that can be expressed in the form $4m - 1$ and in the form $4n + 1$ for integers m and n.

4. (2+4pt)

(a) Let $\mathcal{D}$ be the relation on $\mathbb{Z}$ given by $m\mathcal{D}n \iff m$ divides n. Show that $\mathcal{D}$ is **not** antisymmetric.

(b) Let $\mathcal{R}$ be the relation on $\mathbb{N}$ given by $a\mathcal{R}b \iff 3$ divides $a + 2b$. Show that $\mathcal{R}$ is an equivalence relation.

5. (3pt) Let $\mathcal{R}$ be the relation on $\mathbb{Q}$ given by $\{(x, y) \in \mathbb{Q} \times \mathbb{Q} : x - y \in \mathbb{Z}\}$. Find $\overline{\frac{1}{3}}$ (the equivalence class of $\frac{1}{3}$).

6. (4pt) Define a function $f : \mathbb{N} \to \mathbb{N}$ by $f(1) = 3$, $f(2) = 7$, and $f(n+2) = f(n+1) + f(n)$, for all $n \geq 1$. Show that for all $n \in \mathbb{N}$, $f(n+6) = 4\,f(n+3) + f(n)$.

7. (3+3pt) Let $f : \mathbb{N} \times \mathbb{N} \to \mathbb{N}$ be a function defined by $f(m, n) = 2^{m-1}(2n - 1)$.

(a) Show that f is onto $\mathbb{N}$.

(b) find $f^{-1}(\{12, 35\})$.

8. (3pt) Let $A = \left\{ \dfrac{1}{3k+1} : k \in \mathbb{N} \right\}$. Show that A is countable.

9. (4pt) Let A, B, C, and D be sets so that $A \approx C$ and $B \approx D$. Show that if $A \cap B = C \cap D = \phi$, then $A \cup B \approx C \cup D$.

10. (4+1pt) Let $A = (3, 4) \cup [5, 6) \subseteq \mathbb{R}$.

(a) Show that $A \approx (0, 1)$ **without using** the horizontal line test.

(b) What is the cardinal number of A?