Kuwait University **Mathematics Department**
Final Exam **December 26, 2012**
Math 250 - Foundations of Math **Instructor: A. AlAzemi**

Give full reasons for your answer. State clearly any Theorem you use.

1. (6pt) Answer True or False: Give reasons for your answer.

(a) For the universe of $\mathbb{R}$, $(\forall x)(\exists y)\ [x^2 + y^2 = 1]$.

(b) For the universe of $\mathbb{Q}$, $(\forall x)(\exists! y)\ [x^2 + 1 < y < x^2 + 3]$.

(c) The set $\{(a, b) : a, b \in \mathbb{N} \text{ and } 2a + 3b \leq 10\}$ is finite. If it is true, find its elements.

2. (4pt) Show that for any natural number n, we have $\displaystyle\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$.

3. (2+3pt)

(a) Let $\mathcal{D}$ be the relation on $\mathbb{Z}$ given by $m\mathcal{D}n \iff m$ divides n. Show that $\mathcal{D}$ is **not** antisymmetric.

(b) Let $\mathcal{R}$ be the relation on $\mathbb{N}$ given by $a\mathcal{R}b \iff a$ divides b. Show that $\mathcal{R}$ is a partial order on $\mathbb{N}$.

4. (2+2pt) Suppose that $f : [-1, \infty) \to \mathbb{R}$, given by $f(x) = x^2 + 1$; $g : (-\infty, -1] \to \mathbb{R}$, given by $g(x) = x + 3$; and $h : (-\infty, 0] \to \mathbb{R}$, given by $h(x) = x + 1$.

(a) Is $f \cup g$ a function? Explain.

(b) Is $f \cup h$ a function? Explain.

5. (3+3pt) Let $f : \mathbb{N} \times \mathbb{N} \to \mathbb{N}$ be a function defined by $f(m, n) = 2^{m-1}(2n - 1)$.

(a) Show that f is onto $\mathbb{N}$.

(b) find $f^{-1}(\{21, 36\})$.

6. (4pt) Let $f : A \to B$ be a function. Show that if f is a one-to-one function, then f^{-1} is one-to-one function.

7. (4pt) Let A, B, C, and D be sets so that $A \approx C$ and $B \approx D$. Show that $A \times B \approx C \times D$.

8. (1+3pt)

(a) Define what it means for a set A to be **countable**.

(b) Show that $(0, 6) \approx (2, 3)$.

9. (3pt) Let A be a denumerable set and $x \notin A$. Show that $A \cup \{x\}$ is denumerable.

Bonus Question (2pt):

- For each $k \in \mathbb{N}$, let $A_k = [k, \infty)$. Find $\displaystyle\bigcap_{k \in \mathbb{N}} A_k$ and $\displaystyle\bigcup_{k \in \mathbb{N}} A_k$.