Kuwait University
First Exam
Math 363 - Advanced Linear Algebra

Mathematics Department
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1. (**2+2+1 pts.**) Let $\mathbb{W} = \{\, a + bx + cx^2 \in \mathbb{P}_2(\mathbb{R}) : a + b = c \,\}$.

(a) Show that $\mathbb{W}$ is a subspace of $\mathbb{P}_2(\mathbb{R})$.

(b) Find a basis for $\mathbb{W}$.

(c) What is the dimension of $\mathbb{W}$?

2. (**2+3 pts.**)

(a) If $\beta = \{\, x, y \,\}$ is a basis for a vector space $\mathbb{H}$, show that $\gamma = \{\, ax, x + y \,\}$ is also a basis for $\mathbb{H}$ for any nonzero scalar a.

(b) Show that if $\mathbb{W}_1$ and $\mathbb{W}_2$ are two subspaces of a vector space $\mathbb{V}$, then so is $\mathbb{W}_1 \cap \mathbb{W}_2$.

3. (**2+3 pts.**)

(a) Let $\mathbf{T} : \mathbb{P}_1(\mathbb{R}) \to \mathbb{P}_2(\mathbb{R})$ be a linear for which $\mathbf{T}(x+1) = x^2 - 1$ and $\mathbf{T}(x-1) = x^2 + x$. Evaluate $\mathbf{T}(5x - 1)$.

(b) Let $\mathbf{L} : \mathbb{P}_2(\mathbb{R}) \to M_{2\,x\,2}(\mathbb{R})$ be a linear defined by

$$\mathbf{L}(f(x)) = \begin{pmatrix} f(2) - f(1) & 1 \\ 0 & f(0) \end{pmatrix}.$$

Find a basis for the range of $\mathbf{L}$.

4. (**2+2+1 pts.**) Let $\mathbf{T} : \mathbb{P}_1(\mathbb{R}) \to \mathbb{R}^2$ be a linear defined by $\mathbf{T}(a + bx) = (\, a + b, 3b - a \,)$.

(a) Find a matrix representation for $\mathbf{T}$.

(b) Determine whether $\mathbf{T}$ is one-to-one and onto.

(c) Evaluate $\mathbf{T}(3x + 1)$.

5. (**2+3 pts.**) Let $\mathbf{T} : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear defined by

$$\mathbf{T}(x, y) = (\, x, x + 2y \,).$$

Let $\beta = \{\, (1, 0), (0, 1) \,\}$ and $\gamma = \{\, (1, 2), (1, -1) \,\}$ be two ordered bases for $\mathbb{R}^2$.

(a) Find the change of coordinate matrix $\mathbf{Q}$, that changes γ-coordinates into β-coordinates.

(b) Evaluate $[\mathbf{T}]_\gamma$.